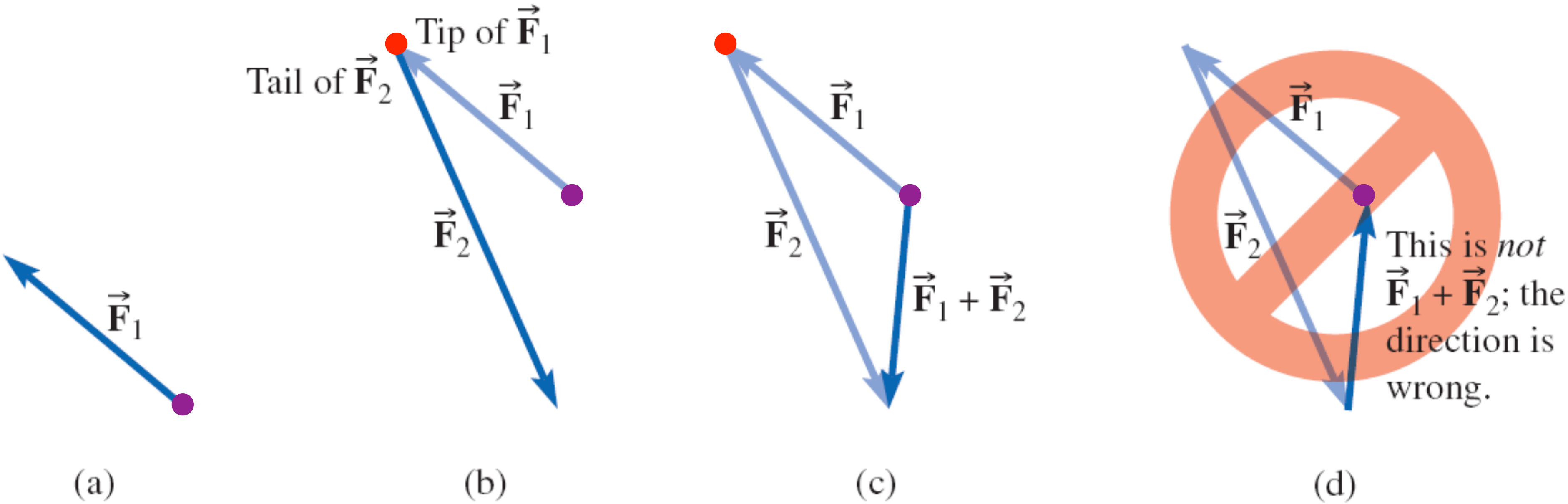
Chapter 3

(Some material taken from McGraw-Hill/College Physics/Giambattista, the majority from your Katz textbook)

Let's begin by discussing how to add two vectors

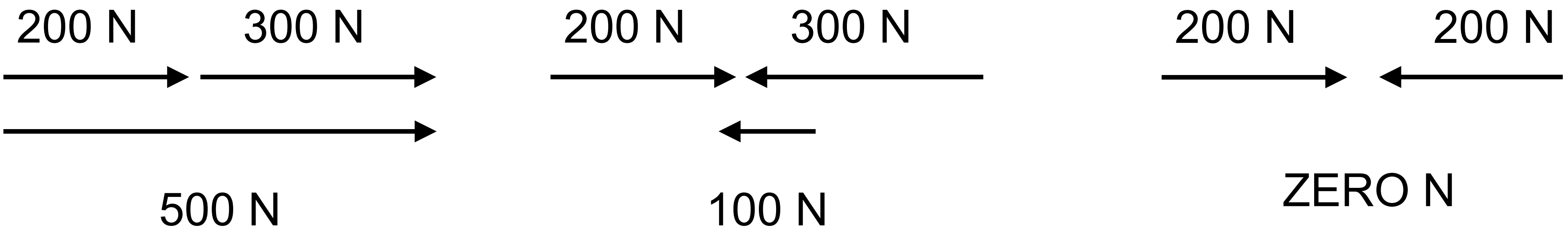
$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$$

The vector goes from where we started to where we end



Let’s calculate the reverse together and see that we get the same answer. Addition of vectors: Order does not matter!

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_1$$



Again, order of addition does not matter

Components of a Vector

Any vector can be expressed as the sum of vectors parallel to the *x*-, *y*-, and (if needed) *z*-axes.

The *x*-, *y*-, and *z*- **components** of a vector indicate the magnitude and direction of the vector along the axes.

A component has magnitude, units, and an algebraic sign (+ or −).

The sign of a component indicates the direction along that axis.

Components of a Vector

The process of finding the components of a vector is called **resolving** the vector into its components.

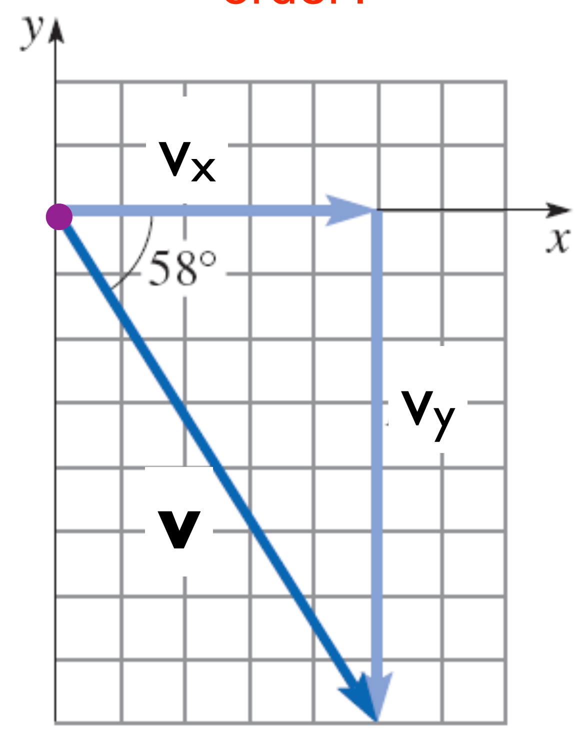
You will be doing a lot of resolving of vectors in the next month! You need to be careful to set up coordinate systems. Normally we use x- and y-axes, but can also use North/West/East/South

Finding Components Consider a velocity vector that has magnitude 9.4 m/s and is directed 58° below the +*x*-axis).

Think of **v** as the sum of two vectors, one parallel to the *x*-axis and the other parallel to the *y*-axis.

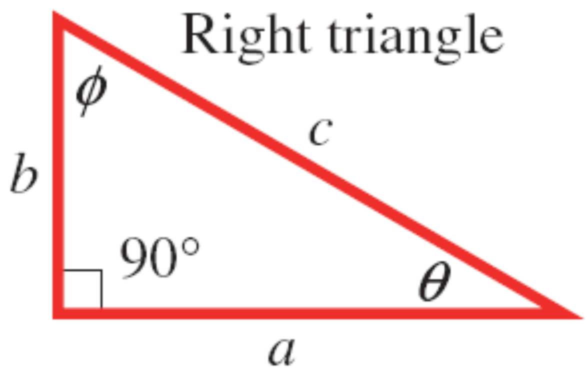
The magnitudes of these two vectors are the *magnitudes* (absolute values) of the *x*- and *y*-components of **v**.

Can you see the addition? Let's talk it over carefully. What happens if we change order?



Anyone here who remembers trigonometry? We will need it!

What about a^2+b^2?



why? $\phi = 90° - \theta$

$$\sin\theta = \frac{\text{side opposite } \angle\theta}{\text{hypotenuse}} = \frac{b}{c}$$

$$\cos\theta = \frac{\text{side adjacent } \angle\theta}{\text{hypotenuse}} = \frac{a}{c}$$

$$\tan\theta = \frac{\text{side opposite } \angle\theta}{\text{side adjacent } \angle\theta} = \frac{b}{a}$$

Some people like to remember SOH–CAH–TOA

What is the sine of zero?
What is the sine of 90 degrees?
What about the cosine of zero?
Cosine 90?

You can always look these up, but by the end of the course you hopefully remember them. No matter what, be careful about radians vs degrees

Robert H. Woodman

@RobertHWoodman

Eating too much cake is the sin of gluttony. However, eating too much pie is okay because the sin of pi is always zero.

9:42 AM · 25 Jul 21 · Twitter Web App

383 Retweets **54** Quote Tweets **1,915** Likes

The sin of pi is zero. (And the sin of eating zero is also zero)

The cos of pi is one. Because when it comes to pi, I want one

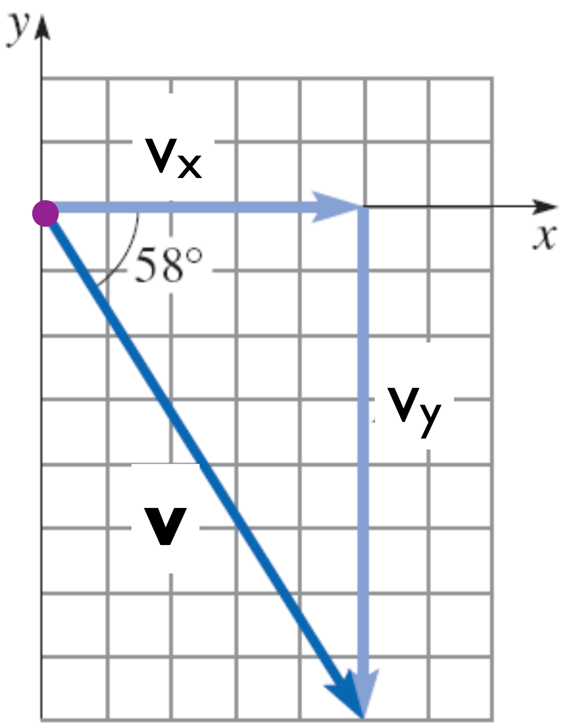
Finding Components

$$\cos 58° = \frac{\text{adjacent}}{\text{hypotenuse}} = |v_x|/v$$

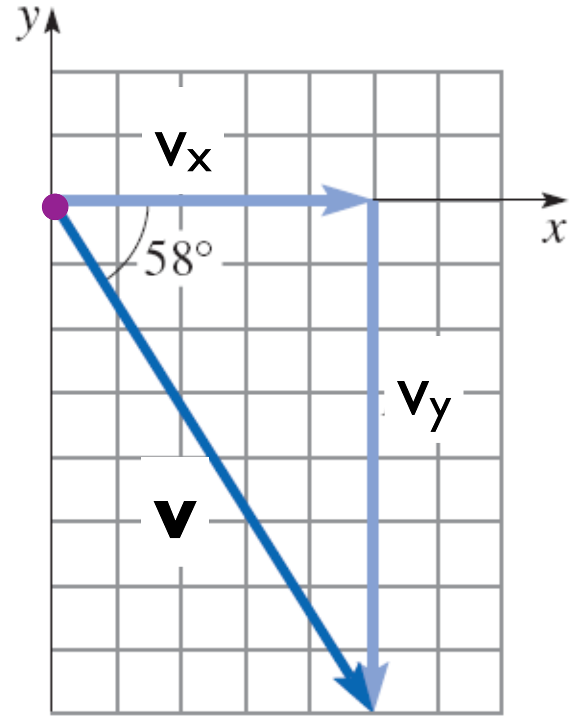
$$\sin 58° = \frac{\text{opposite}}{\text{hypotenuse}} = |v_y|/v$$

$$v_x = +v \cos 58° = 5.0 \text{ m/s}$$

$$v_y = -v \sin 58° = -8.0 \text{ m/s}$$



Note minus sign! Why? Remember that **components** can have sign!



Finding Magnitude and Direction

We can find a vector's magnitude and direction from its components.

$$v_x = +v \cos 58° = 5.0 \text{ m/s}$$

$$v_y = -v \sin 58° = -8.0 \text{ m/s}$$

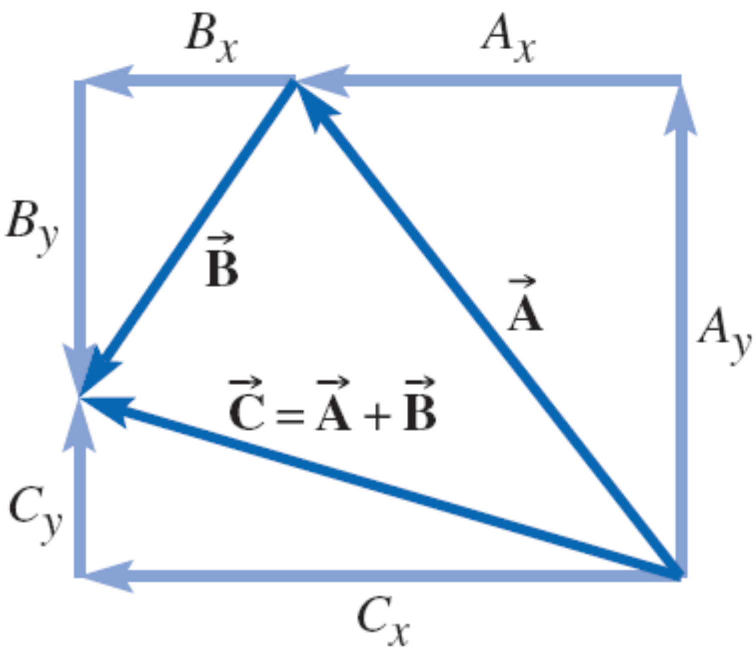
$$\theta = \tan^{-1}\frac{\text{opposite}}{\text{adjacent}} = \tan^{-1}\frac{|v_y|}{|v_x|} = \tan^{-1}\frac{8.0 \text{ m/s}}{5.0 \text{ m/s}} = 58°$$

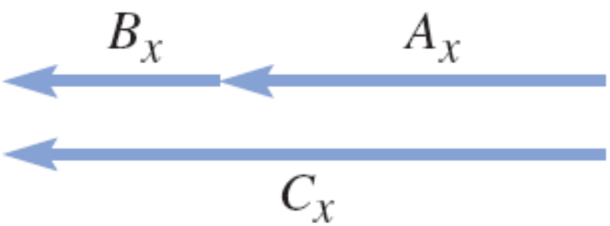
Clear why?

$$|v| = \sqrt{v_x^2 + v_y^2} = \sqrt{(5.0m/s)^2 + (-8.0m/s)^2} = 9.4m/s$$

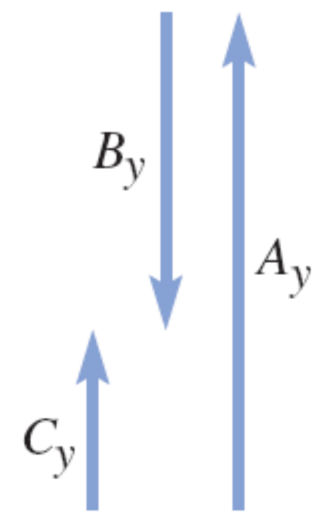
Adding Vectors Using Components

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} \quad \text{if and only if} \quad C_x = A_x + B_x \quad \text{and} \quad C_y = A_y + B_y$$





Let's look careful at these

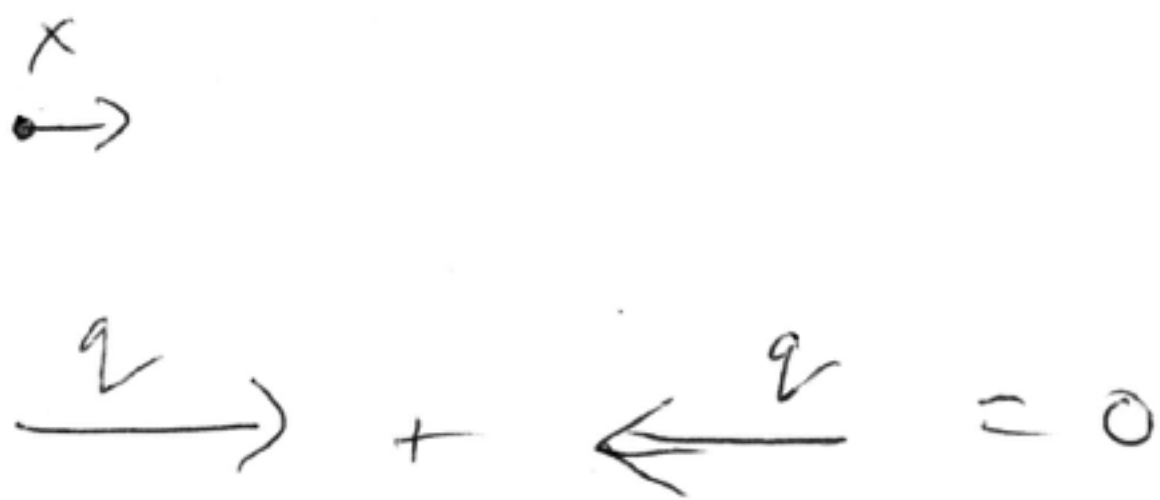


Question for us:

If you have a vector with magnitude q pointing in the +x direction and add to it a vector with magnitude q in the (-x) direction, what is the new vector?

Question for us:

If you have a vector with magnitude q pointing in the +x direction and add to it a vector with magnitude q in the (-x) direction, what is the new vector?



Question for us:

If you have a vector with magnitude q pointing in the +x direction and add to it a vector with magnitude q in the (-y) direction, what is the new vector?

Question for us:

If you have a vector with magnitude q pointing in the +x direction and add to it a vector with magnitude q in the (-y) direction, what is the new vector?

Vector Subtraction

To subtract a vector is to add its opposite (i.e., a vector with the same magnitude but opposite direction):

$$\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i = \vec{\mathbf{r}}_f + (-\vec{\mathbf{r}}_i)$$

Multiplying a vector by the scalar −1 reverses the vector's direction while leaving its magnitude unchanged, so $-\vec{\mathbf{r}}_i = -1 \times \vec{\mathbf{r}}_i$ is a new vector equal in magnitude but in opposite direction

$$\xrightarrow{q} \times -1 = \xleftarrow{-q} \; !$$

What does this look like pictorially?

If my velocity vector is **v**=63 km/hr at 34 degrees north of east, what is **q** = -1 x **v**?

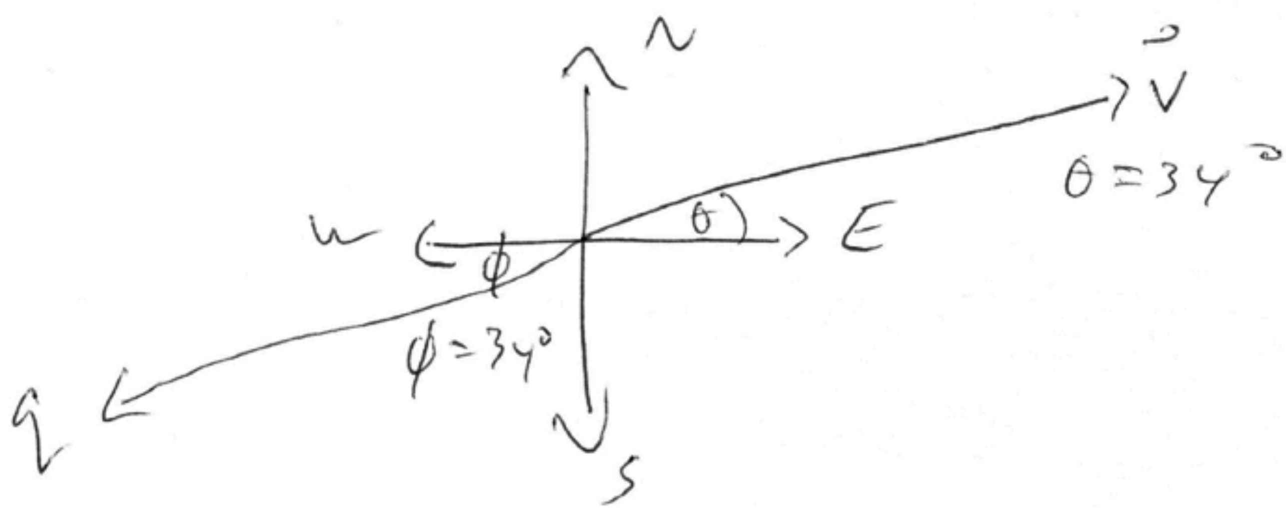
If my velocity vector is **v**=63 km/hr at 34 degrees north of east, what is **q** = -1 x **v**?

If $\vec{v}$ = 63 km/hr 34° N of E then

$\vec{q} = -1 \times \vec{v}$ has the same magnitude as $\vec{v}$

(63 km/hr) & the direction is

reversed, so 34° S of W



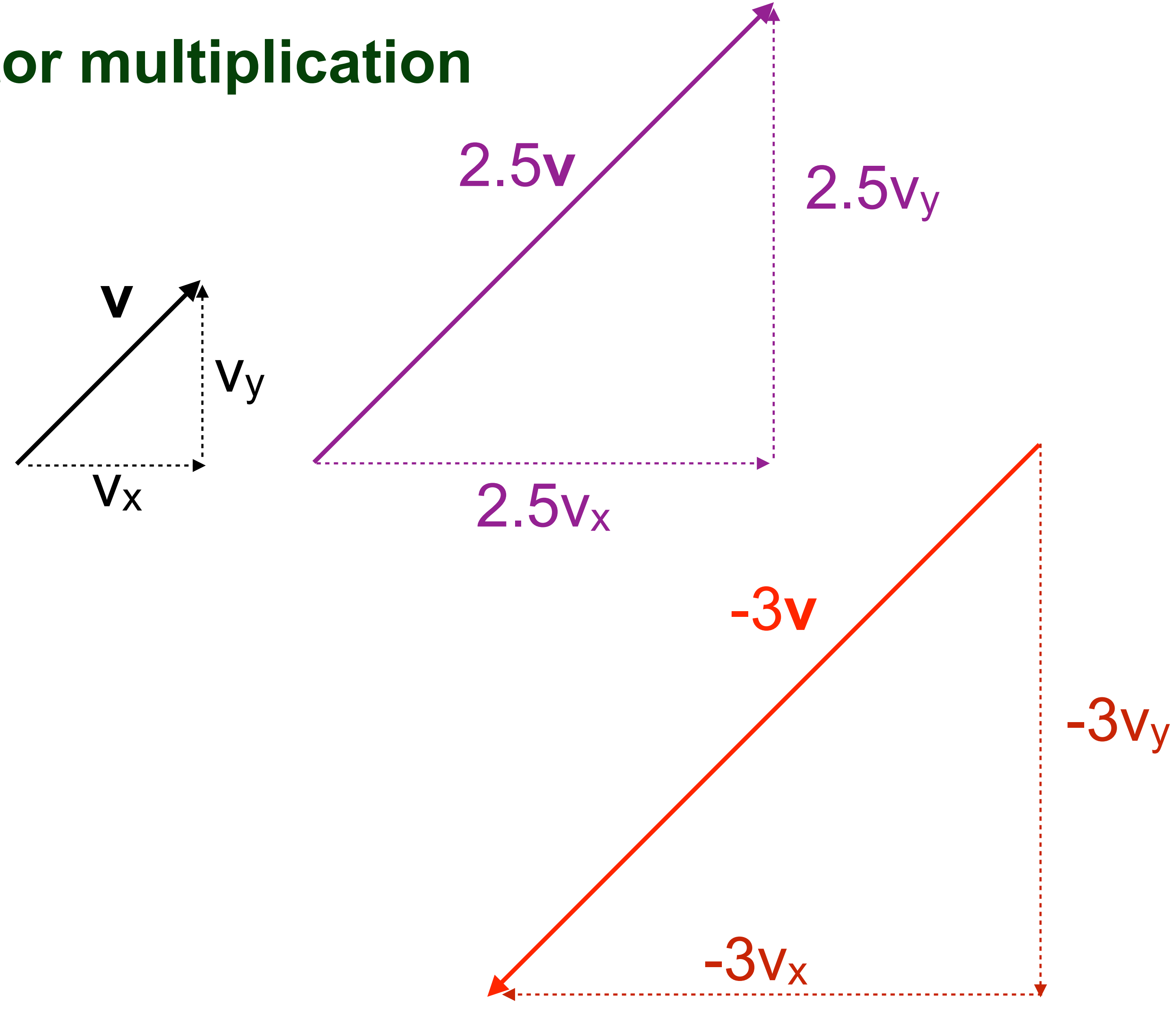
Vector multiplication

We don’t multiply two vectors (we will learn how to do something similar to that at the end of the semester), but we can multiply a scalar and a vector

Multiplying a vector by a scalar keeps the vector in the same direction but changes its length (if the scalar is negative it also reverses the sign). How do we do this with components? We multiply all the components by scalars:

$$s\overrightarrow{A} = s(A_x\hat{x} + A_y\hat{y} + A_z\hat{z}) = sA_x\hat{x} + sA_y\hat{y} + sA_z\hat{z}$$

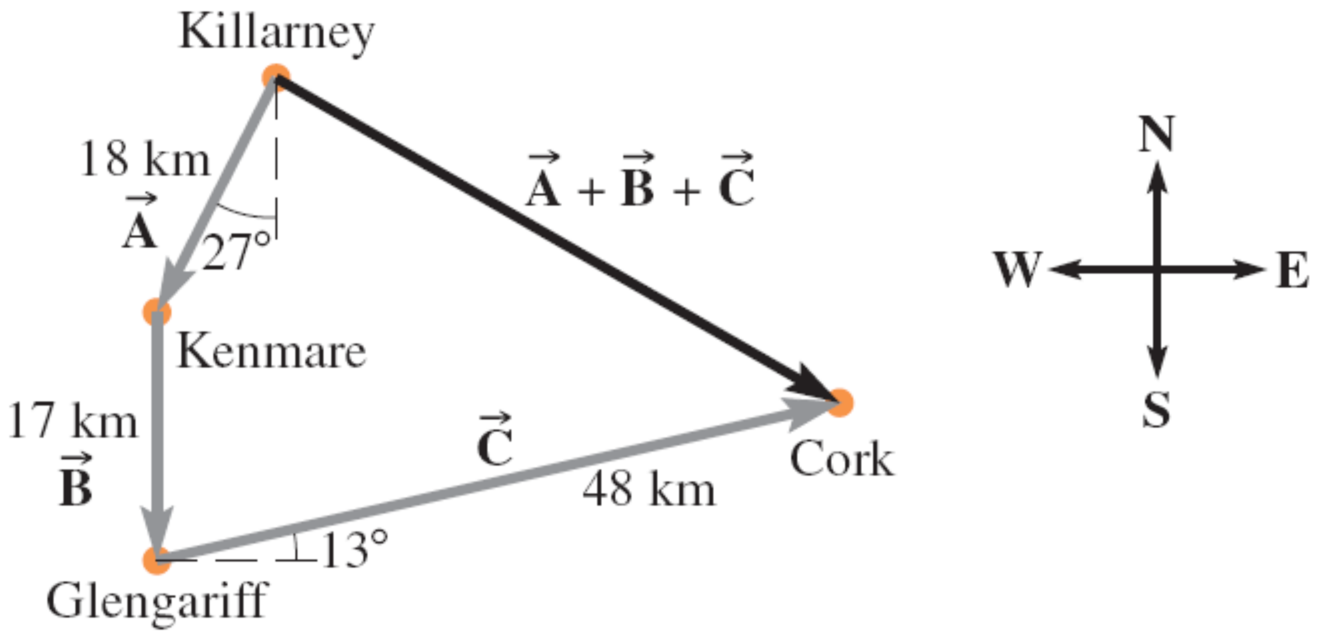
Vector multiplication



In a trip from Killarney to Cork, Charlotte and Shona drive 27° west of south for 18 km to Kenmare, then directly south for 17 km to Lengariff, then 13° north of east for 48 km to Cork.

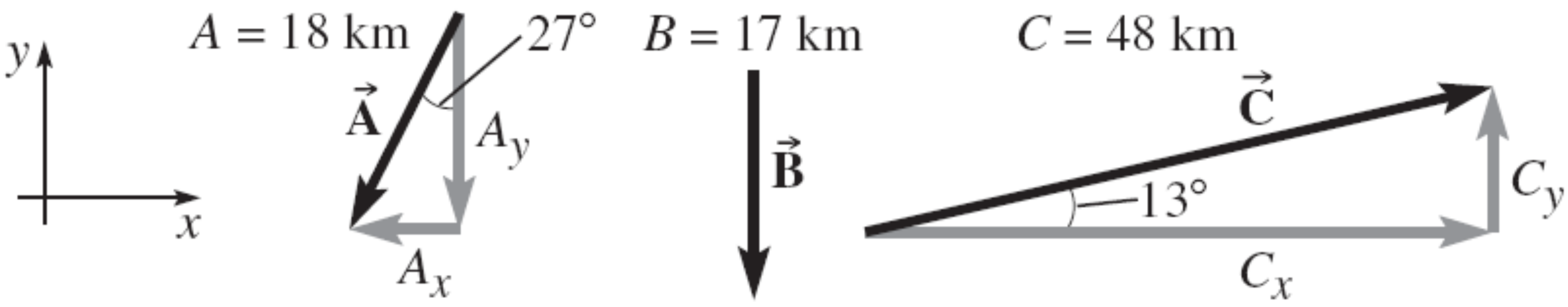
What are the magnitude and direction of the displacement vector for the entire trip?

Coordinate axes and a picture to start



Solution

Live and breathe and die by SOH-CAH-TOA



$$A_x = -A \sin 27° = -18 \text{ km} \times 0.454 = -8.17 \text{ km}$$

$$A_y = -A \cos 27° = -18 \text{ km} \times 0.891 = -16.0 \text{ km}$$

$$B_x = 0 \quad \text{and} \quad B_y = -17 \text{ km}$$

$$C_x = +C \cos 13° = +48 \text{ km} \times 0.974 = +46.8 \text{ km}$$

$$C_y = +C \sin 13° = +48 \text{ km} \times 0.225 = +10.8 \text{ km}$$

Solution

$$\Delta x = A_x + B_x + C_x$$

$$= (-8.17\text{ km}) + 0 + 46.8\text{ km} = 38.63\text{ km}$$

$$\Delta y = A_y + B_y + C_y$$

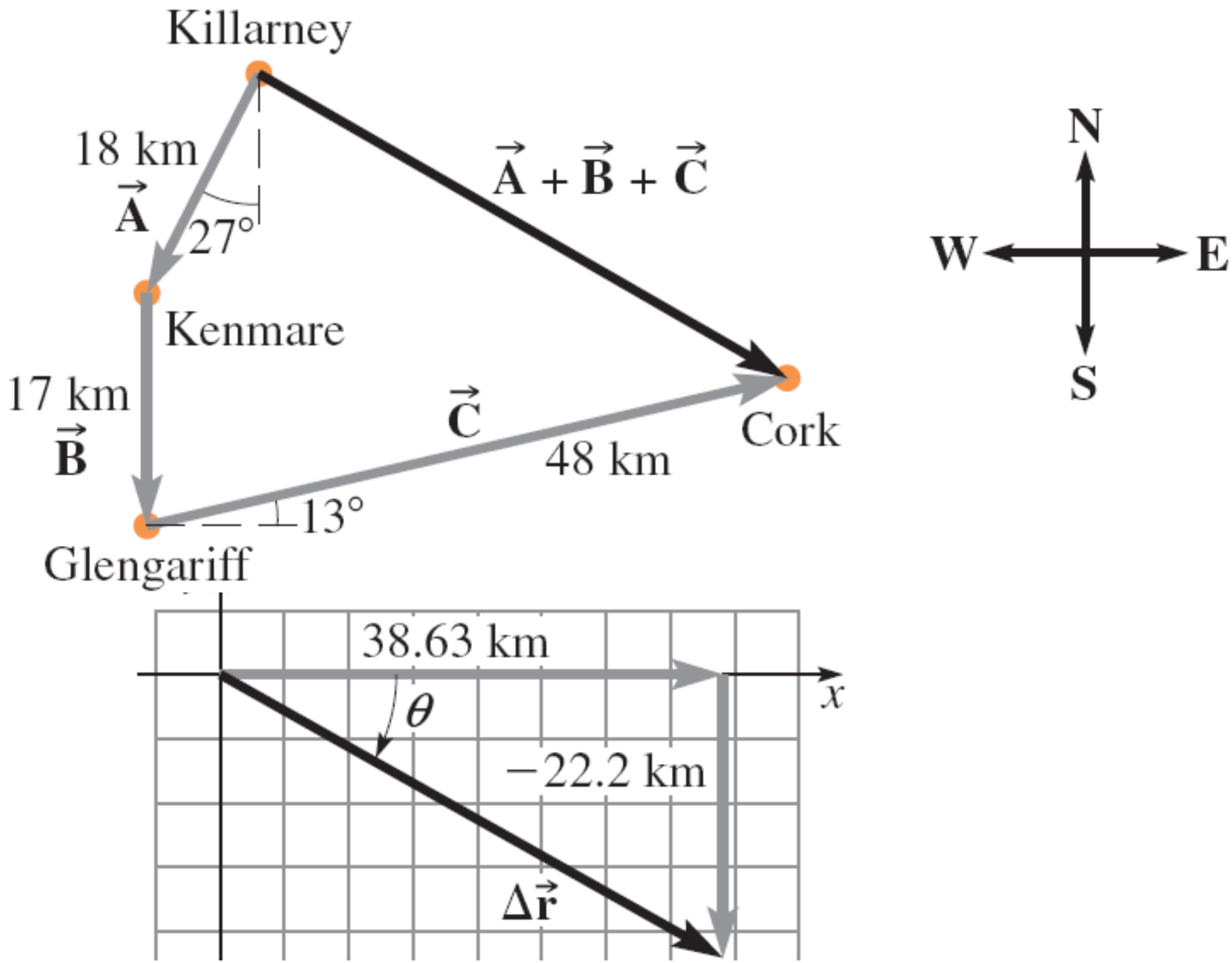
$$= (-16.0\text{ km}) + (-17\text{ km}) + 10.8\text{ km} = -22.2\text{ km}$$

$$\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(38.63\text{ km})^2 + (-22.2\text{ km})^2}$$

$$= 45\text{ km}$$

$$\theta = \tan^{-1}\frac{\text{opposite}}{\text{adjacent}} = \tan^{-1}\frac{22.2\text{ km}}{38.63\text{ km}} = 30^\circ$$

Solution

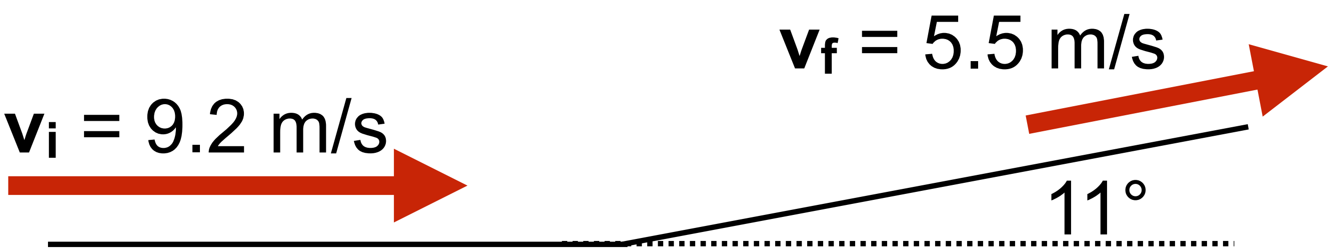

Killarney
18 km
$\vec{A}$
27°
Kenmare
17 km
$\vec{B}$
$\vec{A} + \vec{B} + \vec{C}$
$\vec{C}$
48 km
Cork
13°
Glengariff
N
W
E
S
38.63 km
x
θ
−22.2 km
$\Delta\vec{r}$

A car is traveling on a level, horizontal road with a speed of 9.2 m/s; 150 s later it is climbing a hill with a 11° angle of incline at a speed of 5.5 m/s.

(a) What is the change in the car's speed?

(b) What is the change in the car's velocity?

(c) What is the car's average acceleration during the 150-s time interval?



a) Change in speed
This is the easy one:
Remember, speed is **not** a vector, however to get the right sign: $\Delta(\text{speed}) = \text{speed}_{\text{final}} - \text{speed}_{\text{initial}}$

Change in speed = 5.5 m/s - 9.2 m/s = -3.7 m/s

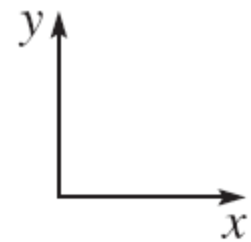
b) Change in velocity

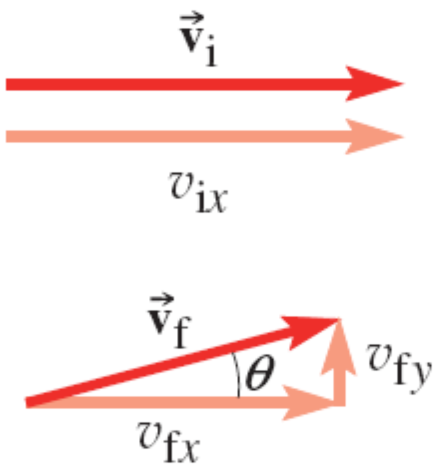
The change in velocity is ***not*** (-3.7 m/s) since we have to account for the change in terms of vectors

The change in velocity is found by subtracting the initial velocity *vector* from the final velocity *vector.*

Solution

(b)





$$v_{iy} = 0, v_{ix} = v_i = 9.2 m/s$$

$$v_{fx} = v_f \cos\theta = 5.5 m/s \cos 11^o = 5.4 m/s$$
$$v_{fy} = v_f \sin\theta = 5.5 m/s \sin 11^o = 1.0 m/s$$

Final-initial works for components too!

$$\Delta v_x = v_{fx} - v_{ix} = -3.8 m/s$$

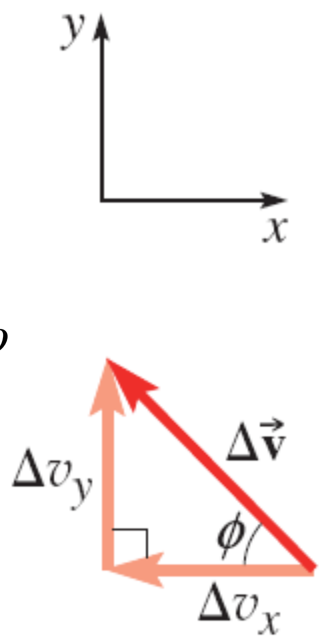
$$\Delta v_y = v_{fy} - v_{iy} = +1.0 m/s$$

Solution

(b)

$$(|\Delta v|^2) = \Delta v_x^2 + \Delta v_y^2 = 15.5m^2/s^2$$

$$|\Delta v| = 3.9m/s$$



$$\tan\phi = \frac{\text{Opp}}{\text{Adj}} = \frac{|\Delta v_y|}{|\Delta v_x|} = 0.28 \rightarrow \phi = 0.27 = 15^o$$

Does the change in direction make sense?

Solution

(c)

$$|a_{av}| = \frac{|\Delta v|}{\Delta t} = \frac{3.9m/s}{150s} = 0.03m/s^2$$

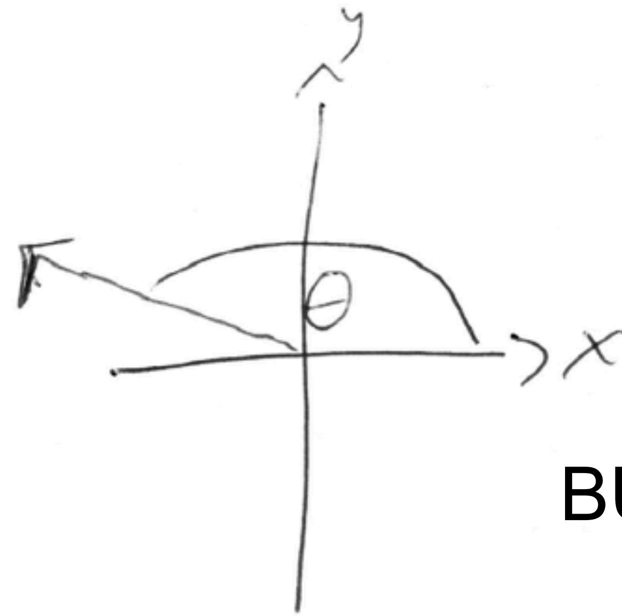
Direction of the average acceleration is the same direction as the change in velocity: 15 degrees clockwise from the negative x axis

Vector **B** has a magnitude of 39.1. Its direction measured counterclockwise from the +x axis is 160.0°. Find its vector components. (Assume that the +x axis is to the right and the +y axis is up along the page. Also, assume the vector is in the x-y plane)

Vector **B** has a magnitude of 39.1. Its direction measured counterclockwise from the +x axis is 160.0°. Find its vector components. (Assume that the +x axis is to the right and the +y axis is up along the page. Also, assume the vector is in the x-y plane)

$|\vec{B}| = 39.1$

$\theta = 160°$



BUT…. (see next slide)

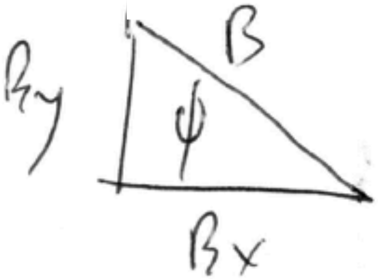
$B_x = |\vec{B}| \cos\theta = -36.7$

$B_y = |\vec{B}| \sin\theta = 13.4$

Vector **B** has a magnitude of 39.1. Its direction measured counterclockwise from the +x axis is 160.0°. Find its vector components. (Assume that the +x axis is to the right and the +y axis is up along the page. Also, assume the vector is in the x-y plane)

$\phi = 180° - \theta = 20°$

Conclusion - ALWAYS check signs (in addition to sines!) when you are dealing with components. Pictures almost always help!



$B_x = B\cos\phi$ but watch for minus sign = - 36.7

$B_y = B\sin\phi$ in (+) y direction = 13.4

GROUP WORK TIME!

https://forms.gle/qZugojd5bpecGaMG9

a) Is it possible for the sum of two vectors to be smaller in magnitude than the magnitude of either vector?

b) Is it possible for the sum of two vectors to be larger than the sum of the magnitudes of the vectors?

The velocity vector of a sprinting cheetah has components v_x = +16.4 m/s and v_y = -26.3 m/s.

a) What is the magnitude of the velocity vector?
b) What angle does the velocity vector make with the x-axis and with the y-axis?

Jerry bicycles from his dorm to the local fitness center: 3.00 miles east and 2.00 miles north. Cindy's apartment is located 1.50 miles west of Jerry's dorm. If Cindy is able to meet Jerry at the fitness center by bicycling in a straight line, what is the distance and direction she must travel?

A speedboat heads west at 108 km/h for 20.0 minutes. It then travels at 60.0 degrees south of west at 90.0 km/h for 10.0 minutes.

a) What is the average speed for the trip?
b) What is the average velocity for the trip?

A particle experiences a constant acceleration that is north at 80 m/s^2. At $t = 0$, its velocity vector is 50 m/s due east.

a) At what time will the magnitude of the velocity be 200 m/s?
b) In what direction is the velocity vector after 3 seconds?

Vectors **A** and **B** are perpendicular and have the same non-zero magnitude. Vector **C** = **A** + **B**, and **D** = **A** - **B**.

a) What is the magnitude of **C**? Sketch the vectors and state your answer
b) If the magnitudes of **C** and **D** are the same, how must **A** and **B** be related? Sketch the vectors and against state your answer

Three vectors **A**, **B** and **C** are related to one another such that 2**A** + 3**B** = 18**C**. A fourth vector **D** exists such that 6**C** + **D** = **A**. Determine an expression for **D** in terms of **A** and **B**.

A truck driver delivering office supplies downtown travels 5.00 blocks north, 3.00 blocks east, and finally 2.00 blocks south.

a) What is the magnitude (in blocks) and the direction of the driver's displacement?
b) What is the total distance traveled (in blocks) by the driver?

A vector’s x component is twice its y component. What angle does the vector make with the x axis?

Vector **B** has a magnitude of 19.5. Its direction measured counterclockwise from the +x axis is 127 degrees. Find its vector components and draw the vector on a coordinate system.

A supertanker begins in Homer, Alaska. It sails 125 km west, and then sails 275 km south. In what direction and for how long would it need to sail to return in a straight line to where it began?

Consider the vector **A** = 16.5 ihat - 33.0 jhat, and **B** = -2.00 ihat + 3.00 jhat - 4.00 khat. Find:

a) **R** = **B**-**A** in component form

b) The magnitude of **R**

Two birds begin next to each other and then fly through the air at the same elevation above level ground at the same speed v (m/s). One flies northeast, and the other flies northwest. After flying for a time t (s), what is the distance measured in meters between them?

Your mother lives 4.4 miles east and 2.2 miles north of you. Your sister lives 1.1 miles west and 3.3 miles south of you. If your sister is able to bike straight from her house to your mother's house, what is the distance and direction she must travel?

Sam rides her bike on a straight line in a direction θ = 32.8° west of south for d = 3.13 miles. What is the distance she would have to ride due south and due west to reach the same location?

Vector **B**, with a magnitude of 23 units, points in the positive y direction. Adding vector **B** to vector **A** yields a resulting vector that points in the negative x direction with a magnitude of 17 units. What are the magnitude and direction of vector **A?**